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Exam Three, MTH 213 , Summer–2022

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Score = $\frac{49}{50}$. V-good

QUESTION 1. Let $A = \{2, 3, 5, 7, -2, -3, -7\}$ Define "=" on A such $\forall\, a, b \in A$ $a" = "b$ if and only if $b = ac$ for some $c \in Z$. Then " = " is an equivalence relation on A.

(i) (5 points) Find all distinct equivalence classes of " = ".

$\bar{2} = \{2, -2\}$ $\bar{7} = \{7, -7\}$

$\bar{3} = \{3, -3\}$

$\bar{5} = \{5\}$

(ii) (5 points) We may view "=" as a subset of $A \times A$. List all elements of " = ".

$" = " = \{(2,2), (2,-2), (-2,2), (-2,-2), (3,3), (3,-3), (-3,3), (-3,-3), (5,5), (7,7), (7,-7), (-7,7), (-7,-7)\}$

QUESTION 2. Let $A = \{2, 8, 3, 6, 12, 18\}$. Define " ≤ " on A such that $\forall\, a, b \in A$, $a" \le "b$ if and only if $a = bc$ for some $c \in \{1, 3, 4, 6, 9\}$. Then " ≤ " is a partial order relation on A.

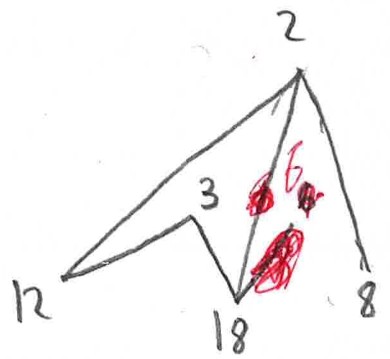
(i) (5 points) Draw the Hasse diagram of " ≤ ".

$2 " \le " 2$ $6 " \le " 6$ $18 " \le " 2$

$8 " \le " 2$ $12 " \le " 2$ $18 " \le " 3$

$8 " \le " 8$ $12 " \le " 3$ $18 " \le " 6$

$3 " \le " 3$ $12 " \le " 12$ $18 " \le " 18$

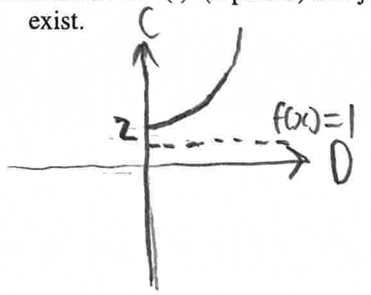


(ii) (5 points) We may view "≤" as a subset of $A \times A$. List all elements of " ≤ ".

$" \le " = \{(2,2), (8,2), (8,8), (3,3), (6,6), (12,2), (12,3), (12,12), (18,2), (18,3), (18,6), (18,18)\}$

(6, 2)

QUESTION 3. (i) (3 points) Let $f : [0, \infty) \to [1, \infty)$ such that $f(x) = x^2 + 2$. Convince me that f^{-1} does not exist.

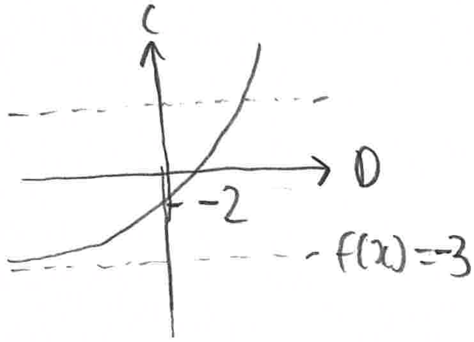


Horizontal line test at $f(x)=1$ does not intersect the line ~~and~~ hence

∴ it is not ~~bijective~~ onto hence f^{-1} does not exist

not bijective

(ii) (5 points) Let $f : R \to (-3, \infty)$ such that $f(x) = e^x - 3$. Convince me that f^{-1} exists. Find the domain and the co-domain of f^{-1}, then find the equation of $f^{-1}(x)$.



Horizontal line test for all $f(x)$ intersects the curve once

∴ it is bijective hence f^{-1} exists

$f^{-1} : (-3, \infty) \to \mathbb{R}$

Domain $= (-3, \infty)$

Codomain $= \mathbb{R}$

$y = e^x - 3$

$y + 3 = e^x$

$x = \ln(y+3)$

$f^{-1}(x) = \ln(x+3)$

(iii) (5 points) Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 3 & 4 & 5 & 1 & 7 & 6 & 9 & 10 & 8 \end{pmatrix}$. Find the least positive integer, n, such that $f^n = f \circ f \circ \cdots \circ f = I$.

$\underbrace{(1\ 2\ 3\ 4\ 5)}_{5\text{-cycle}} \circ \underbrace{(6\ 7)}_{2\text{-cycle}} \circ \underbrace{(8\ 9\ 10)}_{3\text{-cycle}}$

$n = \mathrm{LCM}[5, 2, 3]$

$d = \mathrm{LCM}[5, 2] = \frac{5 \times 2}{\gcd(5,2)} = \frac{10}{1} = 10$

$n = \mathrm{LCM}[10, 3] = \frac{10 \times 3}{\gcd(10,3)} = \frac{30}{1} = 30$

$f^{30} = I$

QUESTION 4. (i) (5 points) There are 1879 persons in a football stadium. Given that the age of each person is between 20 and 23. Then there are at least m persons who are born on the same day of the week, same month, and have the same age. What is the best value of m?

D: 1, ..., 1879

C: 20, ..., 23

$m = \left\lceil \frac{1879}{336} \right\rceil = 6$

$|D| = 1879$

$|C| = 4 \times 12 \times 7 = 336$

(ii) (5 points) there are 257 ODD positive integers. Then there are at least m numbers out of the given 257 odd integers, say $a_1, ..., a_m$, such that $a_1\ (mod\ 8) = a_2\ (mod\ 8) = \ cdots = a_m\ (mod\ 8)$. What is the best value of m?

D: ..., 257

C: 1, 3, 5, 7

$m = \left\lceil \frac{257}{4} \right\rceil = 65$

$|D| = 257$

$|C| = 4$

QUESTION 5. (7 points)

Let $A = \{2, \{2\}, 3, \{2, 3\}\}$. Then write down T or F

(i) $\{2, 3\} \in P(A)$. T

(ii) $\{2, 3\} \in A$. T

(iii) $\{\{2, 3\}, 3\} \in P(A)$ T

(iv) $\{\{2, 3\}, \{3\}\} \subset P(A)$ T

(v) $|P(A)| = 8$ F

(vi) $\{(3, 2), (\{3\}, 2)\} \in P(A \times A)$ F

(vii) $(3, \{2, 3\}) \in A \times A$ T

Faculty information

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